CA Lecture 5c: Digital Logic

Properties of Boolean Algebra

Read Appendix A of textbook: p450 – p456

(Not examined, except DeMorgan theorem)

Properties of Boolean Algebra

	Relationship	Dual	Property
Postulates	$A\,B = B\,A$	$A + B = B + A$	Commutative
	$A\,(B + C) = A\,B + A\,C$	$A + B\,C = (A + B)\,(A + C)$	Distributive
	$1\,A = A$	$0 + A = A$	Identity
	$A\,\overline{A} = 0$	$A + \overline{A} = 1$	Complement
Theorems	$0\,A = 0$	$1 + A = 1$	Zero and one theorems
	$A\,A = A$	$A + A = A$	Idempotence
	$A\,(B\,C) = (A\,B)\,C$	$A + (B + C) = (A + B) + C$	Associative
	$\overline{\overline{A}} = A$		Involution
	$\overline{A\,B} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\,\overline{B}$	DeMorgan's Theorem
	$AB + \overline{A}C + BC = AB + \overline{A}C$	$(A+B)(\overline{A}+C)(B+C) = (A+B)(\overline{A}+C)$	Consensus Theorem
	$A\,(A + B) = A$	$A + A\,B = A$	Absorption Theorem

Properties of Boolean Algebra Cont.

- The postulates are basic axioms of Boolean algebra and therefore need no proofs.
- The theorems can be proven from the postulates.
- Each relationship has both an AND form and an OR form as a result of the **principle of duality**.
- The dual form is obtained by replacing AND with OR and OR with AND, 1's with 0's, and 0's with 1's.

Properties of Boolean Algebra Cont.

- The *commutative* property states that the order that two variables appear in an AND or OR function is not significant.
- The *distributive* property shows how a variable is distributed over an expression.
- The *identity* property states that a variable that is ANDed with 1 or is ORed with 0 produces the original variable.
- The *complement* property states that a variable that is ANDed with its complement is logically false, and a variable that is ORed with its complement is logical true.

Properties of Boolean Algebra Cont.

- The *zero* and *one* theorems state that a variable that is ANDed with 0 produces a 0, and a variable that is ORed with 1 produces a 1.
- The *idempotence* theorem states that a variable that is ANDed or ORed with itself produces the original variable.
- The *associative* theorem states that the order of ANDing or ORing is logically of no consequence.
- The *involution* theorem states that the complement of a complement leaves the original variable (or expression) unchanged.